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Paths containing two adjacent edges in $(2k+1)$-edge-connected graphs

Haruko Okamura

Faculty of Engineering, Osaka City University, Osaka 558, Japan

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Abstract

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1. Introduction

We consider finite undirected graphs, possibly with multiple edges but without loops. Let G be a graph and let $V(G)$ and $E(G)$ be the set of vertices and edges of G, respectively. We allow a repetition of vertices (but not edges) in a path and cycle. For $x, y \in V(G)$, $\lambda(x, y; G)$ denotes the maximal number of edge-disjoint paths between x and y, a path $P = P[x, y]$ denotes a path between x and y, and $\lambda(G) = \min_{x,y \in V(G)} \lambda(x, y; G)$. For $X, Y \subset V(G)$, with $X \cap Y = \emptyset$, $\partial(X, Y; G)$ denotes the set of edges with one end in X and the other in Y, and set $\partial(X; G) = \partial(X, V(G) - X; G)$, $e(X, Y; G) = |\partial(X, Y; G)|$ and $e(X; G) = |\partial(X; G)|$. In the notations, we often omit G. We set $\Gamma(G, k) = \{Z \subset V(G) |$ for each $a, b \in Z$, $\lambda(a, b; G) \geqslant k\}$. For natural numbers $k \geqslant n$, we call a path (or cycle) P in G n-reducible if $\lambda(G) \geqslant k$ and $\lambda(G - E(P)) \geqslant k - n$. If P contains a vertex of degree k as an inner vertex, then P is not 1-reducible. The problem we consider is what kinds of 2-reducible paths and cycles there exist. Lemma 2.1 gives one answer to this problem. For even k, we can find much more 2-reducible paths and cycles. The author has proved the following:

(1.1) (Okamura [4]). If $k \geqslant 4$ is even, $\lambda(G) \geqslant k$, s, t are vertices, and f_1, f_2 are edges, then there is a 2-reducible cycle containing $\{f_1, f_2\}$ and a 2-reducible path $P[s, t]$ containing f_1.

Correspondence to: Haruko Okamura, Faculty of Engineering, Osaka City University, Osaka 558, Japan.

(1.2) (Okamura [6]). Suppose that $k \geqslant 2$ is even, $\lambda(G) \geqslant k$, and s, u, v, t are distinct vertices. Then the following holds:

(a) If $f_1 \in \partial(s,u)$, $f_2 \in \partial(u,v)$, $f_3 \in \partial(v,t)$, and for each $X \subset V(G)$, with $\{f_1, f_2, f_3\} \subset \partial(X)$, $e(X) \geqslant k+2$, then there is a 2-reducible cycle containing $\{f_1, f_2, f_3\}$.

(b) If $f_1 \in \partial(s,u)$, $f_2 \in \partial(u,v) \cup \partial(v,t)$, and for each $X \subset V(G)$, with $X \cap \{s,u,v,t\} = \{s,v\}$, $e(X) \geqslant k+1$, then there is a 2-reducible path $P[s,t]$ containing $\{f_1, f_2\}$.

On the other hand, for each odd $k \geqslant 3$, (1.1) is not always true. In fact, we can construct graphs which contain two vertices x and y such that each cycle containing $\{x,y\}$ is not 2-reducible [8], which together with (1.1), solves a question of Mader [2]. For odd k, (1.2a) is not true in the graph in Fig. 1, and (1.2b) is not true, if $f_2 \in \partial(v,t)$, in the graph in Fig. 2, in which $(k+1)/2$ and $(k-1)/2$ denote the number of parallel edges; however, (1.2b) is true if $f_2 \in \partial(u,v)$, which is our result.

Theorem 1.1. *If $k \geqslant 3$ is odd, $\lambda(G) \geqslant k$, s, u, v, t are distinct vertices, $f_1 \in \partial(s,u)$, $f_2 \in \partial(u,v)$, and for each $X \subset V(G)$, with $X \cap \{s,u,v,t\} = \{s,v\}$, $e(X) \geqslant k+1$, then there is a path $P[s,t]$ containing $\{f_1, f_2\}$ such that $\lambda(G - E(P)) \geqslant k-2$.*

The 2-reducible path problem is itself interesting and also very connected with edge-disjoint paths problem, which poses the question: When do pairwise edge-disjoint paths joining given k pairs of vertices exist? In fact, (1.1) becomes a powerful tool in [5] and [7]. Theorem 1.2 is a generalization of Theorem 1.1, and it seems more useful for the edge-disjoint paths problem.

We call $S \subset V(G)$ dummy if (1.3) below holds.

(1.3) $S = \emptyset$, $\{b\}$ or $\{b, b'\}$, $e(b') = k-1$ and $e(b) \leqslant k-1$ is even.

Theorem 1.2. *Suppose that $k \geqslant 3$ is odd and*

(i) *$V(G) = T \cup W \cup S$ (disjoint union), $T = \{s,u,v,t\}$, S is dummy, $f_1 \in \partial(s,u)$, and $f_2 \in \partial(u,v)$,*

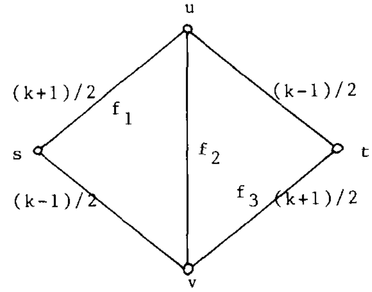

Fig. 1.

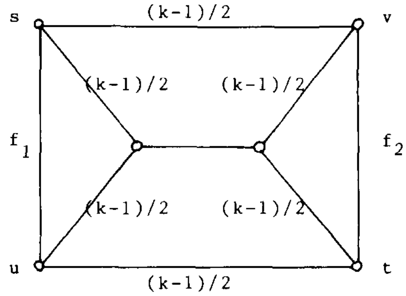

Fig. 2.

(ii) *For each* $x \in T \cup W - \{s, v\}$, $e(x) \geqslant k$, $e(s) \geqslant k-1$, $e(v) \geqslant k-1$, *and* $e(s)$ *or* $e(v) \geqslant k$,
(iii) $V(G) - b \in \Gamma(G, k-1)$, *and* $W^* \in \Gamma(G, k)$ *for* $W^* = \{x \in V(G) | e(x) \geqslant k\}$,
(iv) *For each* $X \subset V(G)$ *with* $X \cap T = \{s, v\}$, $e(X) \geqslant k+1$.
Then there is a path $P[s, t]$ *such that* $\{f_1, f_2\} \subset E(P)$ *and* $W^* \cup \{s\} \in \Gamma(G - E(P), k-2)$.

Let $X \subset V(G)$ and $x \in V(G)$. We set $N(X; G) = \{a \in V(G) - X \mid e(a, x) > 0\}$. G/X denotes the graph obtained from G by contracting X, and, for $a \in X$, we denote the corresponding vertex in G/X by $\tilde{a}$. We call $X \subset V(G)$ a k-set if $|X| \geqslant 2$, $|\bar{X}| \geqslant 2$, and $e(X) = k$, a k-set X is called minimal if for each $Z \subset X$, with $|Z| \geqslant 2$ and $Z \neq X$, $e(Z) \geqslant k+1$. For $a, b \in N(x)$, with $a \neq b$, $f \in \partial(x, a)$ and $g \in \partial(x, b)$, $G_x^{a,b}$ and $G^{f,g}$ denote the graph $(V(G), (E(G) \cup h) - \{f, g\})$, where h is a new edge between a and b, and is called a lifting of G at x arising from the lifting of f and g at x. We call $G_x^{a,b}$ admissible if for each $y, z \in V(G) - x$, with $y \neq z$, $\lambda(y, z; G_x^{a,b}) = \lambda(y, z; G)$.

2. Proof of Theorem 1.2

Lemma 2.1 (Okamura [3]). *Suppose that* $\lambda(G) \geqslant k \geqslant 2$ *and* $\{s, t\} \subset T \in \Gamma(G, k)$. *Then the following holds:*

(1) *If* $f_1, f_2 \in \partial(s)$, *then there is a cycle* C *such that* $f_1, f_2 \in E(C)$ *and* $T \in \Gamma(G - E(C), k-2)$.

(2) *If* $f \in \partial(s)$, *then there is a path* $P[s, t]$ *such that* $f \in E(P)$, $T \in \Gamma(G - E(P), k-2)$, *and* $\lambda(s, t; G - E(P)) \geqslant k-1$.

Lemma 2.2 (Mader [1]). *If* $x \in V(G)$, $e(x) \geqslant 4$, $|N(x)| \geqslant 2$, *and* x *is not a cut-vertex, then there exists an admissible lifting of* G *at* x.

By simple counting, we have the following lemma.

Lemma 2.3. *If* $X, Y \subset V(G)$, *then*

$$e(X-Y) + e(Y-X) = e(X) + e(Y) - 2e(X \cap Y, \overline{X \cup Y}),$$

$$e(X \cap Y) + e(X \cup Y) = e(X) + e(Y) - 2e(X-Y, Y-X).$$

Lemma 2.4. *If* $k \geqslant 3$ *is odd,* $V(G) = X_1 \cup X_2$ *(disjoint union),* $e(X_1) = k+1$, $\lambda(G/X_i) \geqslant k$ $(i = 1, 2)$, *and, for some* $x \in X_1$, $\lambda(x, X_2) = k+1$, *then* $\lambda(G) \geqslant k$.

Proof. Assume that, for some $Y \subset V(G)$, $e(Y) \leqslant k-1$. Then $Y \not\subset X_i$ $(i = 1, 2)$. By Lemma 2.3, we have $e(X_i \cap Y) = k$ and $e(X_i - Y) = k$ $(i = 1, 2)$, contrary to $\lambda(x, X_2) = k+1$. $\square$

We may now assume that G is 2-connected. If $e(u)=d_1>k+1$, then we replace u by d_1-2 vertices of degree k and one vertex of degree $k+1$, and, if $e(v)=d_2>k$, then we replace v by d_2 vertices of degree k, and assign u and v on adjacent new vertices (Fig. 3 gives an example with $d_1=7$, $d_2=6$, and $k=5$), producing a new graph G_1. If the result holds in G_1, then it also holds for G. Thus, we may assume the following:

(2.1) $e(u)\leqslant k+1$, and, for each $x\in W^*-u$, $e(x)=k$.

Now we proceed by induction on $|E(G)|$. We denote the set of required paths by $\mathscr{P}(G,f_1,f_2,t,W^*)$.

(2.2) If $X\subset V(G)$, $|X|\geqslant 2$, $|\bar{X}|\geqslant 2$, *and* $X\cap T\neq\{u,v\}$, $\{s,t\}$, *then* $e(X)\geqslant k+1$.

Proof. If $e(X)=k-1$, then we may let $X\subset V(G)-W^*$, and the result holds in G/X. Thus, $e(X)\geqslant k$, and each vertex in X has even degree, and so $e(X)\geqslant k+1$. Assume $e(X)=k$. If $|X\cap T|=1$, then G/X has a required path P_1. If $X\cap T=\{t\}$, then let $g\in E(P)\cap\partial(X)$. By Lemma 2.1(2), $G/\bar{X}$ has a path $P_2[\bar{s},t]$ such that $g\in E(P_2)$, $(W^*\cap X)\cup\{\bar{s}\}\in\Gamma(G/\bar{X}-E(P_2),k-2)$ and $\lambda(t,\bar{s};G/\bar{X}-E(P_2))=k-1$. Let $P=P_1\cup P_2$ in G. Then by Lemma 2.4, $T\cup W_1\in\Gamma(G-E(P),k-2)$. If $X\cap T=\{s\}$ and $e(s)=k-1$, then for some $x\in X$, $e(x)=k$ and $\lambda(x,\bar{X};G-E(P_1))=k-1$; thus, $W^*\cup\{s\}\in\Gamma(G-E(P_1),k-2)$. If $X\cap T=\{v\}$, then we can extend P_1 to a required path P for G by using Lemma 2.1(1) in $G/\bar{X}$. If $X\cap T=\{s,u\}$, then we can use Lemma 2.1(2) in G/X. $\square$

(2.3) *If* $X\subset V(G)$, $|X|\geqslant 2$, *and* $X\cap T=\{u\}$, *then* $e(X)\geqslant k+2$.

Proof. By (2.2), $e(X)\geqslant k+1$. Assume $e(X)=k+1$. G/X has a required path $P[s,t]$. If $e(u)=k+1$, then $e(u,\bar{X};G-E(P))=k-1$, and, by Lemma 2.4, $W^*\cup\{s\}\in\Gamma(G-E(P),k-2)$. If $e(u)=k$, then for some $x\in X-u$ $e(x)$ is odd; thus, $e(x,\bar{X};G-E(P))=k-1$. $\square$

(2.4) $e(\{u,v\})\geqslant k+1$.

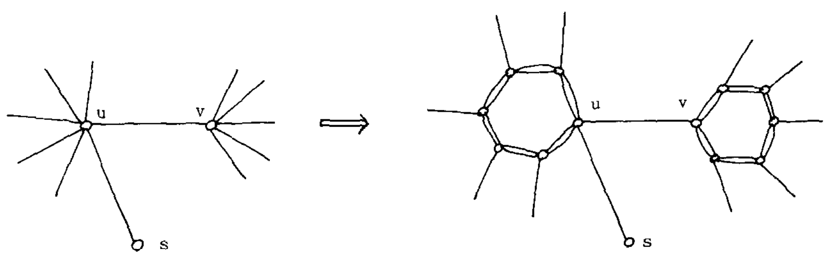

Fig. 3.

Proof. Assume $e(\{u,v\})=k$. Then either $e(u)=k+1$ and $e(v)=k$, or $e(u)=k$ and $e(v)=k-1$. If $N(v)=\{s,v\}$, then $e(\{s,u,v\})=e(s)+e(u)-e(v)-2e(s,u)\leqslant k+1-2=k-1$, a contradiction. Thus, for some $x\in V(G)-\{s,u\}$, *there is an* $h\in\partial(v,x)$. If $x=t$, then the result follows. If $x\neq t$, then, by (2.2), $\mathscr{P}(G/\{u,v\}, f_1, h, t, (W^*-\{u,v\})\cup\{\tilde{u}\})\neq\emptyset$. □

(2.5) $S=\emptyset$.

Proof. Assume $b\in S$, and let $N(b)=\{x_1,\ldots,x_n\}$. Then $n\geqslant 3$, and, by Lemma 2.2, for some $i\neq j$, $G_b^{x_i,x_j}$ is admissible, say for $i=1$ and $j=2$. By (2.1)–(2.3), for each $x\in V(G)$, $e(b,x)<e(b)/2$. Thus, we can choose x_1,x_2 as $x_i\neq b'$ $(i=1,2)$. For $1\leqslant i<j\leqslant n$, let

$$M_{i,j}=\left\{X\subset V(G)-b\;\middle|\;\begin{array}{l}\{x_i,x_j\}\subset X, e(X)\leqslant k+2, \text{and } X\cap T=\{u,t\}\\ \text{or } \{s,v\}.\end{array}\right\}.$$

If $M_{1,2}=\emptyset$, then, by induction, the result holds in $G_b^{x_1,x_2}$; thus, $M_{1,2}\neq\emptyset$. □

(2.5.1) *G has no k-set.*

Proof. For, if G has a k-set Y, then, by (2.2), we may let $Y\cap T=\{s,t\}$. For an $X\in M_{1,2}$, $e(X)=k+2$. If $X\cap T=\{u,t\}$, then, by Lemma 2.3, $e(X\cap Y)+e(\overline{X\cup Y})\leqslant 2k$. $e(X\cap Y)\geqslant k$; so, $e(\overline{X\cup Y})\leqslant k$ and $\overline{X\cup Y}=\{v\}$. Thus, $b\in Y-X$, and, by (2.2), $e(Y-X)\geqslant k+1$. By (2.3) and (2.4), $e(X-Y)\geqslant k+2$, contrary to Lemma 2.3. If $X\cap T=\{s,v\}$, then $e(Y-X\}\geqslant k$. So, $e(X-Y)\leqslant k$, $X-Y=\{v\}$, and, by Lemma 2.3, $e(X\cap Y, \overline{X\cup Y})=1$. Thus, $b\in Y-X$. By (2.3) and (2.4), $e(\overline{X\cup Y})\geqslant k+2$; so, $e(X\cap Y)\leqslant k$ and $X\cap Y=\{s\}$, since $\{x_1,x_2\}=\{s,v\}$, $e(X-Y, Y-X)>0$, contrary to Lemma 2.3. □

We choose $X_1\in M_{1,2}$ such that $|X_1|$ is maximum. We may let $x_3,x_4\notin X_1$, for, if $N(b)-X_1=\{x_3\}$, then $e(b,x_3)=e(b)/2$. We prove the following:

(2.5.2) *If* $G_b^{x_i,x_3}$ $(i=1,2)$ *are not admissible, then* $e(x_i)=k$ $(i=1,2,3)$ *and* $e(x_3,x_i)=(k-1)/2$ (i $=1,2$).

Proof. For $(i,j)=(1,2)$, (2, 1), for some $Y_i\subset V(G)-\{b,x_j\}$, $\{x_i,x_3\}\subset Y_i$, $e(Y_i)=k+1$, and $\partial(Y_i)$ separates W^*. $e(x_i)\geqslant k$ $(i=1$ or $2)$, say for $i=2$. By Lemma 2.3, $e(Y_1-Y_2)+e(Y_2-Y_1)\leqslant 2k$. If $e(Y_1-Y_2)\geqslant k$, then $e(Y_1-Y_2)=e(Y_2-Y_1)=k$; so, by (2.5.1), $Y_1-Y_2=\{x_1\}$ and $Y_2-Y_1=\{x_2\}$. $e(Y_1\cap Y_2)$ and $e(Y_1\cup Y_2)$ are odd; so, $e(Y_1\cup Y_2)\geqslant k+2$ since $G_b^{x_1,x_2}$ is admissible. Thus, by Lemma 2.3, $e(X\cap Y)=k$. Hence, $X\cap Y=\{x_3\}$ and $e(x_i,x_3)=(k-1)/2$ $(i=1,2)$. If $e(Y_1-Y_2)=k-1$, then $Y_1-Y_2=\{x_1\}=\{s\}$ or $\{v\}$, and $e(Y_2-Y_1)=k+1$. Then $e(Y_1\cap Y_2)$ and $e(Y_1\cup Y_2)$ are even. Since $\bar{Y}_2\cap W^*\neq\emptyset$, we have $(\overline{Y_1\cup Y_2})\cap W^*\neq\emptyset$; so, $e(Y_1\cup Y_2)\geqslant k+3$. Thus, $e(Y_1\cap Y_2)=k-1$ and $Y_1\cap Y_2=\{x_3\}=\{b'\}$. Hence, $Y_1\cap W^*=\emptyset$, a contradiction. Now (2.5.2) is proved. □

By (2.5.2), for $i=1$ or 2 and $j=3$ or 4, $G_b^{x_i,x_j}$ is admissible, say for $i=1$ and $j=3$. Let $X_2 \in M_{1,3}$; then $X_1 - X_2 \neq \emptyset$, since $|X_1|$ is maximum.

Case 1: $X_i \cap T = \{s, v\}$ $(i=1,2)$ (or $X_i \cap T = \{u, t\}$ $(i=1,2)$).

By Lemma 2.3, $e(X_1 - X_2) + e(X_2 - X_1) \leqslant 2k + 4 - 2e(u, \{s, v\}) - 2e(b, x_1) = 2k - 2$, contrary to $(X_1 - X_2) \cup (X_2 - X_1) \subset W \cup \{b'\}$.

Case 2: $X_1 \cap T = \{s, v\}$ and $X_2 \cap T = \{u, t\}$.

$e(X_1 - X_2) + e(X_2 - X_1) \leqslant 2k + 4 - 2e(b, x_1) = 2k + 2$; so, $e(X_1 - X_2) = e(X_2 - X_1) = k + 1$. Thus, $e(X_1 \cap X_2)$ and $e(X_1 \cup X_2)$ are odd; so, $e(X_1 \cup X_2) \geqslant k + 2$. Since $e(X_1 \cap X_2) + e(X_1 \cup X_2) \leqslant 2k + 4 - 2e(u, \{s, v\}) = 2k$, $e(X_1 \cap X_2) \leqslant k - 2$, a contradiction. $\square$

(2.6) *The following do not hold*:

(a) *G has a k-set X with* $X \cap T = \{s, t\}$.

(b) *G has a* $(k+1)$*-set Y with* $Y \cap T = \{s, v\}$.

Proof. If both (a) and (b) hold, then $e(X - Y, Y - X) \leqslant 2k - 1$. Thus, $Y - X = \{v\}$, $X - Y = \{t\}$, and $e(v) = k - 1$. By (2.3) and (2.4), $e(\overline{X \cup Y}) \geqslant k + 2$, and $e(X \cap Y) \geqslant k$, a contradiction. $\square$

(2.7) *If* $x_1, x_2 \in W_1$ *and* $g \in \partial(x_1, x_2)$, *then* (a) *with* $g \in \partial(X)$, *or* (b) *with* $g \in \partial(Y)$ *holds.*

Proof. Otherwise, there is a $P \in \mathscr{P}(G - g, f_1, f_2, t, W^* - \{x_1, x_2\})$. If $\{x_1, x_2\} \subset V(P)$, then we replace the subpath of P between x_1 and x_2 by g. $\square$

(2.8) *If* $x_1, x_2 \in W_1$, *then* $e(x_1, x_2) = 0$.

Proof. Assume $g \in \partial(x_1, x_2)$; then, by (2.7), (a) or (b) holds.

Case 1: (a) with $g \in \partial(X)$ holds.

Let X_1 and X_2 be minimal k-sets such that $\{s, t\} \subset X_1 \subset X$ and $\{u, v\} \subset X_2 \subset \bar{X}$. Then the following holds:

(2.8.1) $V(G) = X_1 \cup X_2$.

For, if $X_1 \neq X$, then let Y be a minimal k-set such that $X_1 \subset Y \subset X$ and $X_1 \neq Y$, and let $y_1 \in Y - X_1$ $(\subset W)$. $e(y_1, X_1) \leqslant (k-1)/2$ and $e(y_1, \bar{Y}) \leqslant (k-1)/2$; so, for some $y_2 \in Y - X_1$, there is an $h \in \partial(y_1, y_2)$. Then, by (2.7) and (2.6), there is a k-set Z with $h \in \partial(Z)$ and $Z \cap T = \{s, t\}$. By Lemma 2.3, $e(X_1 \cap Z) = k$ and $\{s, t\} \subset X_1 \cap Z$; so, $X_1 \subset Z$. By Lemma 2.3, $e(Y \cap Z) = k$, $X_1 \subset Y \cap Z \subset Y$ and $X_1 \neq Y \cap Z \neq Y$, contrary to the minimality of Y. Thus, $X_1 = X$ and, similarly, $X_2 = \bar{X}$.

It is easy to see the following:

(2.8.2) If $y_1, y_2 \in X_i \cap W_1$ ($i=1$ or 2), then $e(y_1, y_2)=0$.

We may let $x_i \in X_i$ ($i=1,2$). $N(x_2) \subset X_1 \cup \{u, v\}$ and $e(x_2, X_1) \leqslant (k-1)/2$; so, there is an $h_1 \in \partial(x_2, v)$. By (2.3), $\lambda(G/X_1 - \{f_1, f_2, h_1, g\}) \geqslant k-2$. Now there is an $h_2 \in \partial(x_1, t)$, and $\lambda(G/X_2 - \{f_1, g, h_2\}) \geqslant k-2$ by (2.2).

Case 2. (b) with $g \in \partial(Y)$ holds.

By (2.6) G has no k-set. Let Y_1 and Y_2 be minimal $(k+1)$-sets such that $\{s, v\} \subset Y_1 \subset Y$ and $\{u, t\} \subset Y_2 \subset \bar{Y}$. In the same way as (2.8.1) and (2.8.2) we can prove the following:

(2.8.3) $V(G) = Y_1 \cup Y_2$ and, if $\{y_1, y_2\} \subset Y_i \cap W_1$ ($i=1$ or 2), then $e(y_1, y_2)=0$.

We may let $x_i \in Y_i$ ($i=1,2$). There is an $h_1 \in \partial(x_1, v)$ and $h_2 \in \partial(x_2, t)$. $\lambda(G/Y_2 - \{f_1, f_2, h_1, g\}) \geqslant k-2$ since G has no k-set, and $\lambda(G/Y_1 - \{f_1, f_2, g, h_2\}) \geqslant k-2$ by (2.3). This completes the proof of (2.8). $\square$

Assume that $\{s, u\}$ separates v from t. Since $e(\{u, s\}) \leqslant 2k-1$, we have $e(v) = k-1$, $V(G) = T$, $e(u) = k+1$, and $e(s) = k$. If $e(t, s) \geqslant (k+1)/2$, then $e(\{t, s\}) < k$, and, if $e(t, u) \geqslant (k+1)/2$, then $e(\{u, t\}) \leqslant k$, a contradiction. Thus, $G - \{s, u\}$ has a path $P[v, t]$. By (2.8), $V(P) = \{v, t\}$ or $\{v, x, t\}$ for some $x \in W_1$. If there is an $h \in \partial(v, t)$, then $\lambda(G - \{f_1, f_2, h\}) \geqslant k-2$. If $e(v, t) = 0$, then there are $h_1 \in \partial(v, x)$ and $h_2 \in \partial(x, t)$. By (iv) of the hypothesis and (2.2) and (2.3), $\lambda(G - \{f_1, f_2, h_1, h_2\}) \geqslant k-2$. $\square$

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